1 Electric Stress Estimation and Control

1.1 INTRODUCTION

The potential at a point plays an important role in obtaining any information regarding the electrostatic field at that point. The electric field intensity can be obtained from the potential by gradient operation on the potential

i.e. $$E = -\nabla V \quad ...(1.1)$$

which is nothing but differentiation and the electric field intensity can be used to find electric flux density using the relation

$$D = \varepsilon E \quad ...(1.2)$$

The divergence of this flux density which is again a differentiation results in volume charge density

$$\nabla \cdot D = \rho_v \quad ...(1.3)$$

Therefore, our objective should be to evaluate potential which of course can be found in terms of, charge configuration. However it is not a simple job as the exact distribution of charges for a particular potential at a point is not readily available. Writing $\varepsilon E = D$ in equation (1.3), we have

$$\nabla \cdot \varepsilon E = \rho_v$$

or $$-\nabla \cdot \varepsilon \cdot \nabla V = \rho_v$$

or $$\varepsilon \nabla^2 V = -\rho_v$$

or $$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \quad ...(1.4)$$

This is known as Poisson's equation. However, in most of the high voltage equipments, space charges are not present and hence $\rho_v = 0$ and hence equation (1.4) is written as

$$\nabla^2 V = 0 \quad ...(1.5)$$

Equation (1.5) is known as Laplace's equation.

If $\rho_v = 0$, it indicates zero volume charge density but it allows point charges, line charge, ring charge and surface charge density to exist at singular location as sources of the field.

Here ∇ is a vector operator and is termed as del operator and expressed mathematically in cartesian coordinates as

$$\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z \quad ...(1.6)$$

where $\bar{a}_x, \bar{a}_y$ and $\bar{a}_z$ are unit vectors in the respective increasing directions.

Hence Laplace's equation in cartesian coordinates is given as

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad ...(1.7)$$

Since $\nabla \cdot \nabla$ is a dot product of two vectors, it is a scalar quantity. Following methods are normally used for determination of the potential distribution:

(*i*) Numerical methods

(*ii*) Electrolytic tank method.

Some of the numerical methods used are

(*a*) Finite difference method (FDM)

(*b*) Finite element method (FEM)

(*c*) Charge simulation method (CSM)

(*d*) Surface charge simulation method (SCSM).

1.2 FINITE DIFFERENCE METHOD

Let us assume that voltage variations is a two dimensional problem *i.e.* it varies in *x*-*y* plane and it does not vary along *z*-coordinate and let us divide the interior of a cross-section of the region where the potential distribution is required into squares of length *h* on a side as shown in Fig. 1.1.

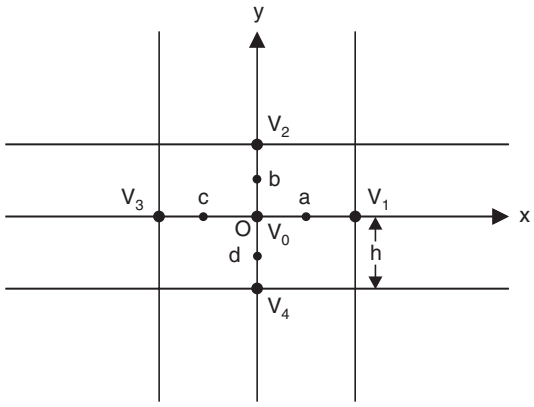


Fig. 1.1 A portion of a region containing a two-dimensional potential field divided into square of side *h*

Assuming the region to be charge free

$$\nabla \cdot D = 0 \quad \text{or} \quad \nabla \cdot E = 0$$

and for a two-dimensional situation

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0$$

and from equation (1.7) the Laplace equation is

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \qquad ...(1.8)$$

Approximate values for these partial derivatives may be obtained in terms of the assumed values. (Here V_0 is to be obtained when V_1, V_2, V_3 and V_4 are known as shown in Fig. 1.1.)

Now $$\left.\frac{\partial V}{\partial x}\right|_a = \frac{V_1 - V_0}{h} \quad \text{and} \quad \left.\frac{\partial V}{\partial x}\right|_c = \frac{V_0 - V_3}{h} \qquad ...(1.9)$$

From the gradients

$$\left.\frac{\partial^2 V}{\partial x^2}\right|_0 = \frac{\left.\frac{\partial V}{\partial x}\right|_a - \left.\frac{\partial V}{\partial x}\right|_c}{h} = \frac{V_1 - V_0 - V_0 + V_3}{h^2} \qquad ...(1.10)$$

Similarly $$\left.\frac{\partial^2 V}{\partial y^2}\right|_0 = \frac{V_2 - V_0 - V_0 + V_4}{h^2}$$

Substituting in equation (1.8), we have

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{V_1 + V_2 + V_3 + V_4 - 4V_0}{h^2} = 0$$

or $$V_0 = \frac{1}{4}(V_1 + V_2 + V_3 + V_4) \qquad ...(1.11)$$

As mentioned earlier the potentials at four corners of the square are either known through computations or at start, these correspond to boundary potentials which are known a priori. From equation (1.11) it is clear that the potential at point *O* is the average of the potential at the four neighbouring points. The iterative method uses equation (1.11) to determine the potential at the corner of every square sub-division in turn and then the process is repeated over the entire region until the difference in values is less than a prespecified value.

The method is found suitable only for two-dimensional symmetrical field where a direct solution is possible. In order to work for irregular three-dimensional field so that these nodes are fixed upon boundaries, becomes extremely difficult. Also to solve for such fields as very large number of $V(x, y)$ values of potential are required which needs very large computer memory and computation time and hence this method is normally not recommended for a solution of such electrostatic problems.

1.3 FINITE ELEMENT METHOD

This method is not based on seeking the direct solution of Laplace equation as in case of FDM, instead in Finite element method use is made of the fact that in an electrostatic field the total energy enclosed in the whole field region acquires a minimum value. This means that this voltage distribution under given conditions of electrode surface should make the enclosed energy function to be a minimum for a given dielectric volume v.

We know that electrostatic energy stored per unit volume is given as

$$W = \frac{1}{2}\,\varepsilon\, E^2 \qquad ...(1.12)$$

For a situation where electric field is not uniform, and if it can be assumed uniform for a differential volume δv, the electric energy over the complete volume is given as

$$W = \int_V \frac{1}{2}\,\varepsilon\,(-\nabla V)^2\, dv \qquad ...(1.13)$$

To obtain voltage distribution, our performance index is to minimise W as given in equation (1.13).

Let us assume an isotropic dielectric medium and an electrostatic field without any space charge. The potential V would be determined by the boundaries formed by the metal electrode surfaces.

Equation (1.13) can be rewritten in cartesian coordinates as

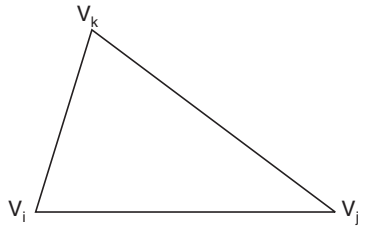
$$W = \frac{1}{2}\,\varepsilon \iiint \left[\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] dxdydz \qquad ...(1.14)$$

Assuming that potential distribution is only two-dimensional and there is no change in potential along z-direction, then $\frac{\partial V}{\partial z} = 0$ and hence equation (1.14) reduces to

$$W_A = z \iint \left[\frac{1}{2}\,\varepsilon \left\{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2\right\}\right] dxdy \qquad ...(1.15)$$

Here z is constant and W_A represents the energy density per unit area and the quantity within integral sign represents differential energy per elementary area $dA = dxdy$.

In this method also the field between electrodes is divided into discrete elements as in FDM. The shape of these elements is chosen to be triangular for two dimensional representation and tetrahedron for three-dimensional field representation Fig. 1.2 (*a*) and (*b*).



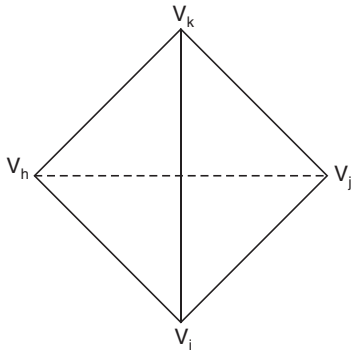


Fig. 1.2 (*a*) Triangular finite element e_1 (*b*) Tetrahedron finite element

The shape and size of these finite elements is suitably chosen and these are irregularly distributed within the field. It is to be noted that wherever within the medium higher electric stresses are expected *e.g.* corners and edges of electrodes, triangles of smaller size should be chosen.

Let us consider an element e_1 as shown in Fig. 1.2(*a*) as part of the total field having nodes i, j and k in anticlockwise direction. There will be a large no. of such elements $e_1, e_2 \ldots .e_N$. Having obtained the potential of the nodes of these elements, the potential distribution within each elements is required to be obtained. For this normally a linear relations of V on x and y is assumed and hence the first order approximation gives

$$V(x, y) = a_1 + a_2 x + a_3 y \qquad ...(1.16)$$

It is to be noted that for better accuracy of results higher order approximation *e.g.* square or cubic would be required. Equation (1.16) implies that electric field intensity within the element is constant and potentials at any point within the element are linearly distributed. The potentials at nodes i, j and k are given as

$$V_i = a_1 + a_2 x_i + a_3 y_i$$
$$V_j = a_1 + a_2 x_j + a_3 y_j$$
$$V_k = a_1 + a_2 x_k + a_3 y_k \qquad ...(1.17)$$

Equation (1.17) can be rewritten in matrix form as

$$\begin{bmatrix} V_i \\ V_j \\ V_k \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad ...(1.18)$$

By using Cramer's rules, the coefficient a_1, a_2, a_3 can be obtained as follows

$$a_1 = \frac{1}{2\Delta_e}(\alpha_i V_i + \alpha_j V_j + \alpha_k V_k) \qquad ...(1.19)$$

$$a_2 = \frac{1}{2\Delta_e}(\beta_i V_i + \beta_j V_j + \beta_k V_k)$$

and $$a_3 = \frac{1}{2\Delta_e}(\gamma_i V_i + \gamma_j V_j + \gamma_k V_k)$$

where $\alpha_i = x_j y_k - x_k y_j$, $\alpha_j = x_k y_i - x_i y_k$, $\alpha_k = x_i y_j - x_j y_i$

$\beta_i = y_j - y_k$, $\beta_j = y_k - y_i$, $\beta_k = y_i - y_j$

$\gamma_i = x_k - x_j$, $\gamma_j = x_i - x_k$, $\gamma_k = x_j - x_i$

and $$2\Delta_e = \alpha_i + \alpha_j + \alpha_k = \beta_i \gamma_j - \beta_j \gamma_i$$

where Δ_e represents the area of the triangular element under consideration. As mentioned earlier the nodes must be numbered anticlockwise, else Δ_e may turn out to be negative.

From equation (1.16), the partial derivatives of V are

$$\frac{\partial V}{\partial x} = a_2 = f(V_i, V_j, V_k) \quad \text{and} \quad \frac{\partial V}{\partial y} = a_3 = f(V_i, V_j, V_k) \qquad ...(1.20)$$

We know that for obtaining the voltage at various nodes we have to minimise the energy within the whole system for which derivatives of energies with respect to potential distribution in each element is required. For the element e under consideration, let W_e be the energy enclosed in the element, then energy per unit length in the z-direction W_e/z denoted by $W_{\Delta e}$ can be obtained by using equation (1.15) as follows

$$W_{\Delta e} = \frac{W_e}{z} = \frac{1}{2} \varepsilon \Delta_e \left\{ \left(\frac{\partial V}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 \right\}_e \qquad ...(1.21)$$

Here $$\Delta_e = \iint_e dxdy$$

To obtain condition for energy minimisation we differentiate partially equation (1.21) with respect to V_i, V_j and V_k separately. Thus partially differentiating equation (1.21) with respect to V_i and making use of equations (1.19) and (1.20).

We have $$\frac{\partial W_{\Delta e}}{\partial V_i} = \frac{1}{2}\,\varepsilon\,\Delta_e\left(2a_2\,\frac{\partial a_2}{\partial V_i} + \frac{\partial a_3}{\partial V_i}\right) = \frac{1}{2}\,\varepsilon\,(a_2\beta_i + a_3\gamma_i)$$

$$= \frac{\varepsilon}{4\Delta_e}[(\beta_i^2 + \gamma_i^2)\,V_i + (\beta_i\beta_j + \gamma_i\gamma_j)V_j + (\beta_i\beta_k + \gamma_i\gamma_k)V_k] \quad ...(1.22)$$

Similarly, finding partial derivatives of equation (1.21) with respect to V_j and V_k and following the procedure outlined above for partial derivative with respect to V_i and arranging all the three equations in matrix from we have

$$\frac{\partial W_{\Delta e}}{\partial V_e} = \frac{\varepsilon}{4\Delta_e}\begin{bmatrix}(\beta_i^2 + \gamma_i^2) & (\beta_i\beta_j + \gamma_i\gamma_j) & (\beta_i\beta_k + \gamma_i\gamma_k)\\ (\beta_j\beta_i + \gamma_j\gamma_i) & (\beta_j^2 + \gamma_j^2) & (\beta_j\beta_k + \gamma_j\gamma_k)\\ (\beta_k\beta_i + \gamma_k\gamma_i) & (\beta_k\beta_j + \gamma_k\gamma_j) & (\beta_k^2 + \gamma_k^2)\end{bmatrix}_e \begin{bmatrix}V_i\\ V_j\\ V_k\end{bmatrix}_e \quad ...(1.22a)$$

$$= \frac{\varepsilon}{4\Delta_e}\begin{bmatrix}C_{ii} & C_{ij} & C_{ik}\\ C_{ji} & C_{jj} & C_{jk}\\ C_{ki} & C_{kj} & C_{kk}\end{bmatrix}_e \begin{bmatrix}V_i\\ V_j\\ V_k\end{bmatrix}_e = [C]_e\,[V]_e \quad ...(1.23)$$

After considering a typical element e, the next step is to take into account all such elements in the region under consideration and the energy associated with all the elements will then be

$$W = \sum_{e=1}^{N} W_e = \frac{1}{2}\,\varepsilon\,[V^T]\,[C]\,[V] \quad ...(1.24)$$

where $$[V] = \begin{bmatrix}V_1\\ V_2\\ \vdots\\ V_n\end{bmatrix}$$

and n is the total number of nodes in the system and N is the no. of elements and $[C]$ is called the global stiffness matrix which is the sum of the individual matrices.

In general $\dfrac{\partial W}{\partial V_h}$ leads to

$$\sum_{i=1}^{n} V_i C_{ik} = 0 \quad ...(1.25)$$

The solution of the above equations gives voltage distribution in the region. Of course while seeking the final solution the boundary conditions must be satisfied and hence this would require some iterative method for the exact solution.

The second approach could be to formulate energy function in terms of the unknown nodal voltage. This energy function is subjected to certain constraints in terms of boundary conditions. The objective then is to min. $[W]$ subject to certain constraints. For this various mathematical programming techniques like, Fletcher Powell technique, Fletcher technique, direct search techniques, self scaling variable metric techniques can be used. A computer program can be developed and accuracy of the result can be obtained depending upon the convergence criterion fed into the computer. A suitable initial guess for the solution can always be made depending upon the system configuration and during every iteration the voltage can be updated till all the boundary conditions are satisfied and the energy

function is minimised that is when the change in the energy function between two consecutive iterations is less than a prespecified value.

The finite element method is useful for estimating electric fields at highly curved and thin electrode surfaces with composite dielectric materials especially when the electric fields are uniform or weakly non-uniform and can be expressed in two-dimensional geometries. The method is normally not recommended for three-dimensional non-uniform fields.

1.4 CHARGE SIMULATION METHOD

As suggested by the name itself, in this method, the distributed charges on the surface of a conductor/electrode or dielectric interfaces is simulated by replacing these charges by n discrete fictitious individual charges arranged suitably inside the conductor or outside the space in which the field is to be computed. These charges could be in the form of point, line or ring, depending upon the shape of the electrode under consideration. It could be a suitable combination of these fictitious charges. The position and type of simulation charges are to be determined first and then the field on the electrode surface is determined by the potential function of these individual charges. In order to determine the magnitude of these charges n no. of points are chosen on the surface of the conductor. These points are known as "contour points''. The sum of the potentials due to fictitious charge distribution at any contour points should correspond to the conductor potential V_c which is known a priori.

Suppose q_i, is one of the fictitious charges and V_i is the potential of any point P_i in space which is independent of the coordinate system chosen, the total potential V_i due to all the charges is given as

$$V_i = \sum_{j=1}^{n} p_{ij} q_j \qquad ...(1.26)$$

where p_{ij} are known as ''potential coefficient'' which are to be determined for different types of charges by using Laplace's equation. We know that potential at a point P at a distance 'a' from a point charge q is given as

$$V = \frac{q}{4\pi\varepsilon a} \qquad ...(1.27)$$

So here the potential coefficient p is $\frac{1}{4\pi\varepsilon a}$.

Similarly, these coefficients for linear and ring or circular charges can also be obtained. It is found these are also dependent upon various distance of these charges from the point under consideration where potential is to be obtained and the permittivity of the medium as in case of a point charge and hence potential coefficients are constant number and hence the potential due to various types of charges are a linear function of charges and this is how we get the potential at a point due to various charges as an algebraic sum of potential due to individual charges.

A few contour points must also be taken at the electrode boundaries also and the potential due to the simulated charge system should be obtained at these points and this should correspond to the equipotentials or else, the type and location of charges should be changed to acquire the desired shape and the given potential. Suppose we take 'n' number of contour points and n no. of charges, the following set of equations can be written as

$$\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & & & \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \quad ...(1.28)$$

The solution of these equations gives the magnitude of the individual charges and which corresponds to electrode potential (V_1 ..... V_n) at the given discrete points. Next, it is necessary to check whether the type and location of charges as obtained from the solution of equation (1.28) satisfies the actual boundary conditions every where on the electrode surfaces. It is just possible that at certain check points the charges may not satisfy the potential at those points. This check for individual point is carried out using equation (1.26). If simulation does not meet the accuracy criterion, the procedure is repeated by changing either the number or type or location or all, of the simulation charges till adequate charge system (simulation) is obtained. Once, this is achieved, potential or electric field intensity at any point can be obtained.

The field intensity at a point due to various charges is obtained by vector addition of intensity due to individual charges at that point. However, it is desirable to obtain the individual directional components of field intensity separately. In cartesion coordinate system, the component of electric field intensity along x-direction for n number of charges is given as

$$\overline{E}_x = \sum_{j=1}^{n} \frac{\partial p_{ij}}{\partial x} q_j = \sum_{j=1}^{n} (f_{ij})_x \, q_j \quad ...(1.29)$$

where $(f_{ij})_x$ are known as field intensity coefficients in x-direction.

In this method it is very important to select a suitable type of simulation charges and their location for faster convergence of the solution *e.g.* for cylindrical electrodes finite line charges are suitable, spherical electrodes have point charges or ring charges as suitable charges. However, for fields with axial symmetry having projected circular structures, ring charges are found better. Experience of working on such problems certainly will play an important role for better and faster selection. The procedure for CSM is summarised as follows:

1. Choose a suitable type and location of simulation charges within the electrode system.
2. Select some contour point on the surface of the electrodes. A relatively larger no. of contour points should be selected on the curved or corner points of the electrode.
3. Calculate the p_{ij} for different charges and locations (contour points) and assemble in the form of a matrix.
4. Obtain inverse of this matrix and calculate the magnitude of charges (simulation).
5. Test whether the solution so obtained is feasible or not by selecting some check points on the conductor surface. If the solution is feasible stop and calculate the electric field intensity at requisite point. If not, repeat the procedure by either changing the type or location of the simulation charges.

CSM has proved quite useful for estimation of electric field intensity for two and three-dimensional fields both with or without axial symmetry. It is a simple method and is found computationally efficient and provides accurate results.

The simplicity with which CSM takes care of curved and rounded surfaces of electrodes or interfaces of composite dielectric medium makes it a suitable method for field estimation. The computation time is much less as compared to FDM and FEM.

However, it is difficult to apply this methods for thin electrodes *e.g.* foils, plates or coatings as some minimum gap distance between the location of a charge and electrode contours is required. Also, it is found difficult to apply this method for electrodes with highly irregular and complicated boundaries with sharp edges etc.

However, as mentioned earlier a good experience of selecting type and location of simulation charge may solve some of these problem.

An improved version of CSM known as surface charge simulation method (SCSM) described below is used to overcome the problem faced in CSM.

1.5 SURFACE CHARGE SIMULATION METHOD

Here a suitably distributed surface charge is used to simulate the complete equipotential surface *i.e.* the electrode contour since the surface charge is located on the contour surface itself. In actual practice the existing surface charge on the electrode configuration is simulated by integration of ring charges placed on the electrode contour and dielectric boundaries. This results into a physically correct reproduction of the whole electrode configuration.

The electrode contours are segmented as shown in Fig. 1.3 and to each segment '*S*' a surface charge density is assigned by a given function $S_k(x)$ which could be a first degree approximation or a polynomial as follows

$$\sigma(x) = \sum_{k=0}^{n} S_k(x).\sigma_k \qquad ...(1.29a)$$

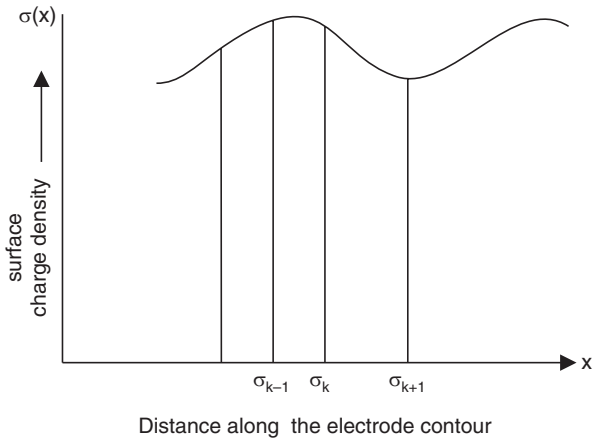


Fig. 1.3 Segmented contour path with assigned σ

The individual segments along the contour path can be represented as shown in Fig. 1.4.

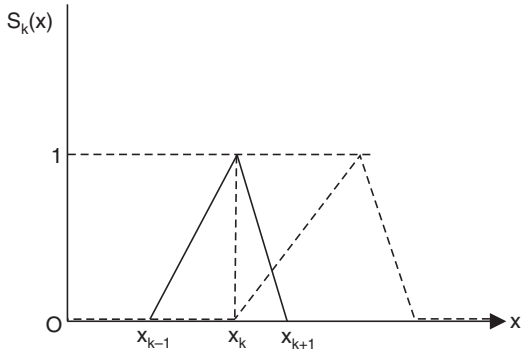


Fig. 1.4 Representation of a segment $S_k(x)$

The value of $S_k(x)$ is zero for $x < x_{k-1}$ and is unity at $x = x_k$ and in between x_{k-1} and x_k is given as $\frac{x - x_{k-1}}{x_k - x_{k-1}}$.

With the representation the contour surface is reproduced accurately and exactly and thus the continuity of charge between the segments is assumed. Surface charges can be simulated either by line or ring charges. Ring charge simulation is found to be more useful for fields with symmetry of rotation. Each contour segment is assigned m no. of charges and the potential due to a charge q_j, is given by equation (1.26) and is rewritten here

$$V_i = \sum_{j=1}^{m} p_i q_j$$

The potential coefficient p_{ik} for a contour point i due to kth contour segment is obtained as shown in Fig. 1.5 and is given as

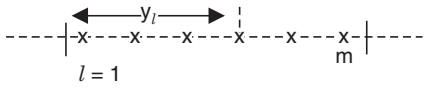


Fig. 1.5 Concentrated charges to simulate surface charges

$$p_{ik}\,\sigma_k = \int_x \sigma(x) \cdot p_{ix}\, dx \qquad ...(1.30)$$

Now substituting equation (1.29 *a*) in equation (1.30), we have

$$p_{ik}\sigma_k = \int_x \sum_{k=0}^{n} S_k(x) \cdot \sigma_k p_{ix}\, dx \qquad ...(1.31)$$

Since each segment is divided into m intervals as shown in Fig. 1.5, equation (1.31) can be rewritten as

$$p_{ik}\sigma_k = \int_{x_{k-1}}^{xk} S_k(x)\, p_{ix}\, dx = \sum_{l=1}^{m} S_k(y_l)\, p_{il}\sigma_k \qquad ...(1.32)$$

The potential coefficient p_{il} are similar to the coefficients derived from a single concentrated charge in CSM. This coefficient, therefore, can be obtained for a line charge or by solving elliptical integral for a ring charge. The electric field intensity at any contour point i due to kth contour segment is given as

$$E_i = \sum_{k=1}^{m} \sigma_k \, f_{ik} \qquad ...(1.33)$$

where f_{ik} are the field intensity coefficients.

As discussed this method requires a large number of elements, normally more than 2500, independent of the surface shape and thus require large computational efforts. Also, due to certain practical difficulties this method is not used as frequently as other numerical methods for estimation of electric fields.

1.6 COMPARISON OF VARIOUS TECHNIQUES

Out of the various techniques FDM is the simplest to compute and understand but the computation effort and computer memory requirements are the highest. Also, since all difference equations are approximation to the actual field conditions, the final solution may have considerable error.

Finite element method is a general method and has been used for almost all fields of engineering. The method is suitable for estimating fields at highly curved and thin electrode surfaces with different dielectric materials. However, this method is more useful for uniform or weakly non-uniform fields and which can be represented by two-dimensional geometries. This method is recommended for three-dimensional complicated field configurations.

Charge Simulation Method (CSM) is considered to be one of the most superior and acceptable method for two and three-dimensional configuration with more than one dielectric and with electrode systems of any desired shape since this method is based on minimization of the energy function which could be subjected to any operating constraints *e.g.* environmental condition, it has proved to be highly accurate method. Because of inherent features of the technique, this method also helps in optimising electrode configuration. In this electrode configuration optimisation problems the objective is to have field intensity as low as possible subject to the condition that a constant field intensity exists on the complete electrode surface. With this optimisation, a higher life expectancy of high voltage equipments can be achieved.

However, as mentioned earlier this method can not be used for thin electrodes *e.g.* foils, plates or coatings due to the requirement of a minimum gap distance between the location of a charge and electrode contour. Also, this method is not suitable for highly irregular electrode boundaries.

The surface charge simulation method even though takes into account the actual surface charge distribution on the electrode surface, this method is not normally recommended for solution of field problem due to some practical difficulties.

An important difference between the various method is that the FDM and FEM can be used only for bounded field whereas CSM and SCMS can also be used for unbounded fields.

1.7 ELECTROLYTIC TANK

For assessing electric field distribution in complex three-dimensional situations, analytical methods are unsuitable. Two other approaches in use are, experimental analog and numbrical techniques. The numerical techniques have already been discussed in the previous section. We now study analog techniques especially the use of electrolytic tank.

The potential distribution in conductive media in current equilibrium condition satisfies Laplace's equation the same as the electric fields in space-charge-free regions *i.e.* $\rho_v = 0$. This fact makes it possible to obtain solutions to many difficult electrostatic field problems by constructing an analogous potential distribution in a conductive medium where the potential and field distributions can be measured directly. The conductors and insulation arrangements can be represented using an electrolytic tank. Due to its simplicity and accuracy this method has been used for decades.

A scale model of the electrode configuration is set up in a tank with insulating walls, filled with suitable electrolyte *e.g.* tap water. An alternating voltage is the appropriate choice of working voltage to avoid polarisation voltage arising in the case of direct voltages. The equipotential lines or equipotential areas in the case of the electric field are measured by means of a probe which can be fed with different voltages from a potential divider *via* a null indicator.

Guiding the probe along the lines corresponding to the potential selected on the divider as well as their graphical representation, can be undertaken manually or automatically in large systems. For the two-dimensional field model, various dielectric constants can be simulated by different heights of electrolyte as shown in Fig. 1.6 for a cylinder-plane configuration.

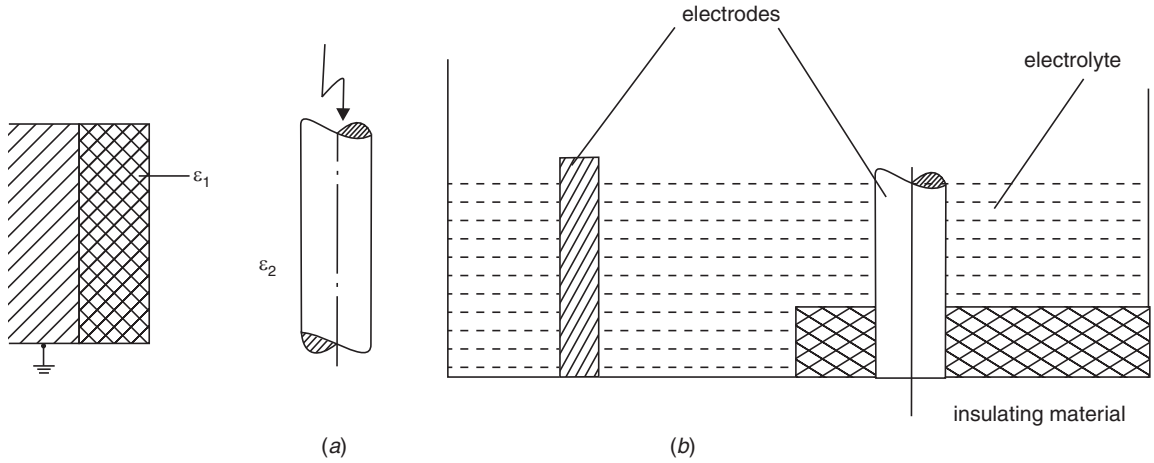


Fig. 1.6 Simulation of a cylinder-plane configuration in the electrolytic tank
(*a*) Original (*b*) Simulation for the case $\varepsilon_1 = 2\varepsilon_2$

Three-dimensional fields with rotational symmetry can be readily simulated in a wedge shaped tank whereas for fields with no rotational symmetry one has to resort to much more complex forms of three-dimensional simulation.

Equipotential boundaries are represented in the tank by specially formed sheets of metal. For example a single dielectric problem such as a three core cable may be represented using a flat tank as shown in Fig. 1.7. Different permittivities are represented by electrolytes of different conductivities separated by special partitions. Otherwise, the tank base can be specially shaped.

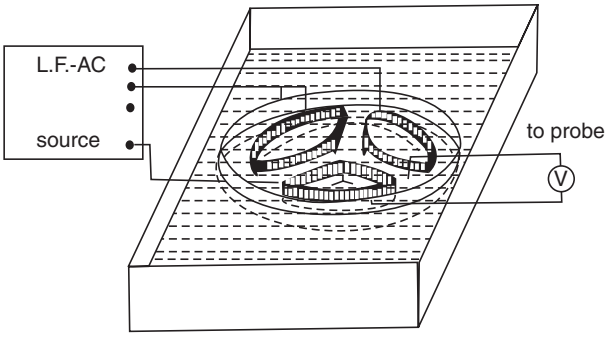


Fig. 1.7 Electrolytic tank model of a three-core cable represented at the instant when one core is at zero voltage, the same as the sheath

1.8 CONTROL OF ELECTRIC FIELD INTENSITY

It is a common knowledge that if the field in a dielectric material is uniform, the material is properly utilised. If it is non-uniform the material is under-utilised. Under normal situation an electric field is not uniform due to imperfection in the dielectric material during manufacture or it could be due to undesirable shapes and sizes of electrodes.

Sudden change in shape of electrodes in the form of corners or edges in high voltage equipments leads to concentration of electric fields at such locations resulting in higher electric stresses on the dielectric. The area around such location becomes highly vulnerable to breakdown of insulation. In order to avoid this breakdown, the electrodes should be suitably designed and shaped so that concentration of the field is not allowed. The electrodes are extended and so shaped that a higher field intensity than the main field does not appear anywhere in the dielectric material. To achieve this objective Rogowaski suggested a shape by which electrodes should be extended known as Rogowaski profile as shown in Fig. 1.8(*b*).

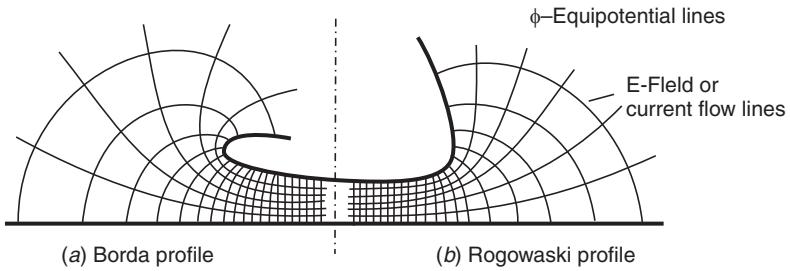


Fig. 1.8 Equipotential and the field (current flow) lines between plane and brim field

It is clear from the Fig. 1.8 that the electric field intensity continuously reduces beyond the edges of the electrodes. Another profile of electrodes suggested by Borda for reducing electric field stresses is shown in Fig. 1.8(*a*) . Comparing the two profiles, it is found that the Borda profile achieves a lower field intensity beyond the edges as compared to Rogowaski profile.

Also, it is found that the space requirements for high voltage equipments is smaller with Borda profile electrodes as compared to Rogowaski profile. In many situations for high voltage equipment

space requirements become a serious problem and hence electric field optimisation techniques have received a great importance.

A visit to a high voltage laboratory shows that electrodes at high potential are given large, smooth shaped dome like shapes to bring down the electric field stress surrounding the area *i.e.* the atmospheric air. The modern trend is to design segmented electrodes in which a number of small, identical, smooth discs are given a desired continuous shapes as per requirement.

Electric stress control shields of various shapes and sizes are used for high voltage equipments as shown in Fig. 1.9. Sometimes sharp electrode ends are enclosed by a large diameter hemispherical electrode having a smooth aperture.

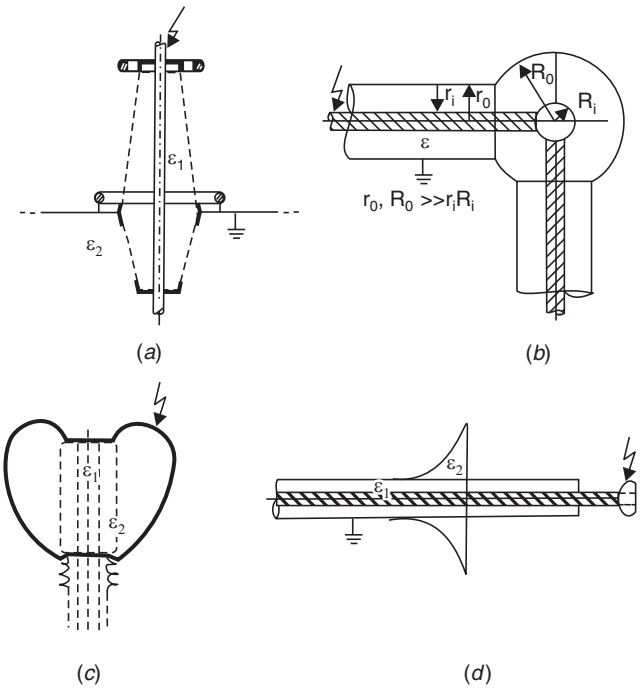


Fig. 1.9 Extended shapes of electrodes for stress control (*a*) A bushing with toroids (*b*) Right angle turn of a bus bar in gas insulated switchgear (*GIS*), (*c*) HV electrode on a condenser, (*d*) Stress cone on a screened cable end

Circular and tubular electrodes are provided with spherical shields at bends. In high voltage laboratories tubular electrodes of large diameter are used for connection rather than insulated wire thereby electric field intensity is reduced considerably.

For transmission voltage 400 kV and above we make use of more than one conductor which is known as bundling of conductors or these conductors are known as bundled conductors. This is done to reduce electric field intensity in the atmospheric air thereby the corona loss on the line and radio interference with communication lines is reduced.

The design of high voltage bushing used for large capacity power transformers, potential transformers or cable termination is based on capacitance grading thereby a uniform voltage distribution is achieved which results in a uniform electric field intensity within the dielectric. This is done by

interposing concentric sheets of metals of suitable lengths and position. This is known as "inter-sheath grading" to control electric field intensity. The inter-sheaths are held at suitable potential which enables economic utilisation of the insulating material by evenly distributing the equipotential surfaces.

A simple rule to control electric field intensity in high voltage equipments is to avoid sharp points and edges in the electrodes. These electrodes should be large symmetrical and should have smooth surface. The surface should not be rough as this would lead to higher stresses at high voltages. Micro protrusion on the surface of the electrode may penetrate deeper into the dielectric material which may result in high electric field intensity at those points and may lead to breakdown of the dielectric material.

1.9 OPTIMISATION OF ELECTRODE CONFIGURATION

Various numerical techniques have been used to optimise the electrode configuration so that the dielectric material is optimally utilised as a result a considerable improvement in dielectric behaviour is achieved and a higher life expectancy of high voltage equipments can be anticipated. When we talk of electrode configuration optimisation, we really mean the electric field intensity optimisation. Even though some work has been dedicated for electrode configurations optimisation by FDM and FEM methods, yet the inherent suitability of CSM for optimisation, lot of work has been reported in literature using this technique.

The objective of optimisation is to determine the configuration of electrodes which may result into a minimum and constant field intensity on the complete electrode surface. The optimisation technique is based on the partial discharge inception electric field intensity E_{pd} which depends upon the dielectric material, its pressure (if gas is the medium) and the electrode configuration. It is to be noted that if the electric field is uniform or weakly non-uniform, the partial discharge or normal breakdown takes place at the same electric field intensity. Therefore, it is only the electrode configuration which can be optimised. If E_{pd} is more than the electric field intensity E applied, partial discharge can not take place which means the electrode can be said to be optimised, if at a given voltage the maximum value of $\frac{E}{E_{pd}}$ on its surface is as small as possible. Since the maximum value of E/E_{pd} depends upon three parametres the shape, size and position of electrodes, three different types of optimisation possibilities exist. The optimum shape of an electrode is characterised by

$$\text{Min. } (E/E_{pd})_{\max} = \text{constant} \qquad \ldots(1.34)$$

The optimisation methods are based on iterative process and when equation (1.34) is satisfied, the optimum electrode configuration is obtained. While using CSM, following strategies are used for optimisation of electrode configuration:

(*i*) Displacement of contour points perpendicular to the surface

(*ii*) Changing the position of the ''optimisation charges'' and contour points

(*iii*) Modification of contour elements

A brief view of these methods is given below.

1.9.1 Displacement of Contour Points

In this method a constant magnitude of electric field intensity is achieved using an iterative process by differential displacement of contour points perpendicular to the surface during every iteration. We start with a suitable contour of the electrode and on this we fix some contour points, and, electric field intensity is evaluated for this contour. If this field intensity is within the permissible value of desired field intensity we stop as the optimal configuration is obtained right at the first step. However, it is very unlikely to hit at the optimal configuration in the first step itself and hence the curvature of this contour is changed step by step depending upon the difference between the calculated electric field intensity and the desired field intensity. A good experience in field theory will be quite helpful in deciding the new contour points during different iterations. Figure 1.10 shows the flow chart for the optimisation process.

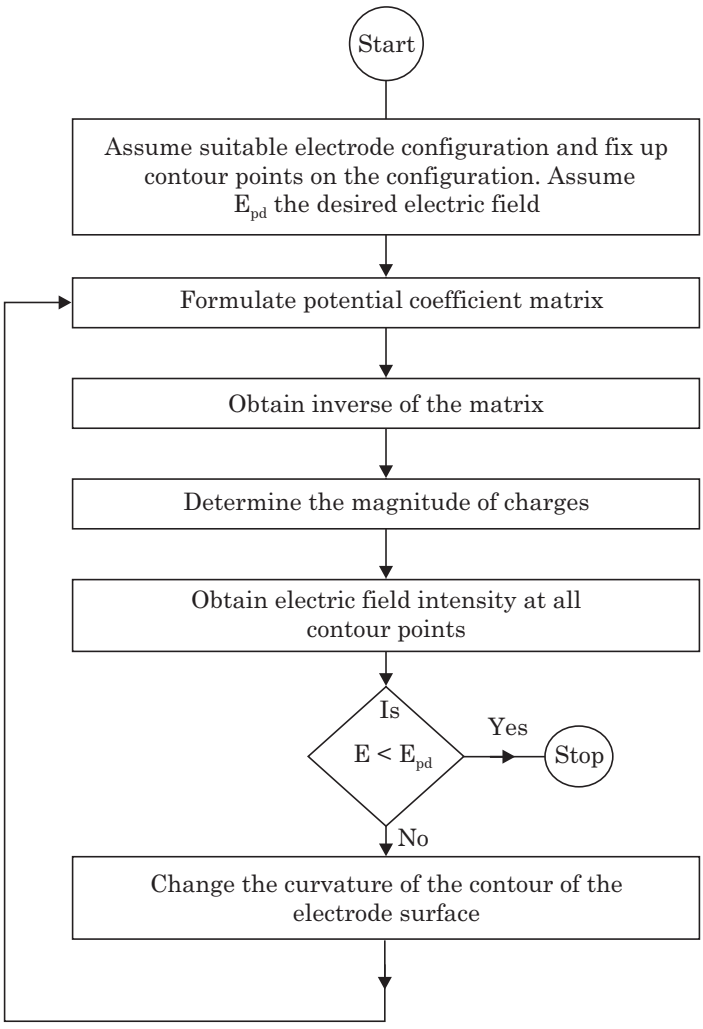


Fig. 1.10 Flow chart for optimisation by displacement of contour points method

It is to be noted that for electric field calculation on the electrode surface (assumed or updated) is caried out using CSM technique. Since during every iteration the complete CSM technique is to be used for calculation of electric field intensity, computer time requirement is high and this is its major disadvantage.

1.9.2 Changing the Position of the Optimisation Charges and Contour Points

In CSM we start with certain configuration of the electrode and we calculate electrode field intensity at various contour points of the electrode and from this we should be able to identity which part of the electrode surface needs optimisation (the contour points where $E > E_{pd}$). Normally in CSM, the identified region of electrode configuration is reproduced by a set of contour points and a set of simulation charges at fixed locations but of unknown magnitudes. However, in optimisation method, the optimisation region is reproduced like in CSM by a suitable 'n' number of contour optimisation points as shown in Fig. 1.11.

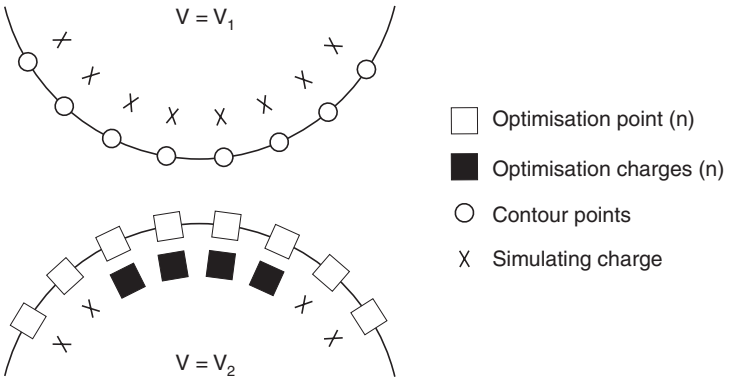


Fig. 1.11 Optimisation region in an electrode configuration

This optimisation region is then provided with 'n' optimisation charges which have fixed position and known magnitudes. The optimisation charges are obtained corresponding to a partial solution of mathematical problem.

The optimisation procedure proceeds with iterative steps as follows:

(*i*) The optimisation charges which affect the field intensity in optimisation region more than the simulation charges are shifted in position such that the field intensity in the optimisation region is less than a prespecified value.

(*ii*) When we shift the optimisation charges the potential at the contour points in the region is changed. In order to bring the known potential (new) at the contour points, the magnitude of the simulation charges should be changed to fulfil the new potential requirement at the contour points.

(*iii*) The new equipotential line having the known potential is calculated with the help of a complete new set of optimisation charges. This will be our new optimised contour of the electrode.

(*iv*) The field intensity in the optimisation region is calculated and compared it with a prespecified value of E and this may not satisfy equation (1.34) or even after changing the shape of the electrode and the charge magnitudes in the other region.

(*v*) The optimisation points are now located at the newly obtained contour of the electrode. The location of the respective optimisation charges should, therefore, be suitably corrected to start the next iteration.

The iteration procedure is continued with lower magnitude of electric field intensity till it converges to a prespecified value or instruct to stop the computation process if it goes to a very low value which is physically not feasible.

1.9.3 Modification of Contour Elements

This method is based on the qualitative correlation between the curvature of an electrode surface and its electric field intensity. The larger the curvature the larger is the value E/E_{pd}. Therefore, the radii of the contour element will have to be increased for the region where E/E_{pd} is large and decreased where E/E_{pd} is small. The optimisation method is explained here for a cylindrical rod and plane electrode configuration. It is required to optimise the contour of the rod end. Suppose *r* is the radius of the rod end and '*d*' is its distance from the plane as shown in Fig. 1.12(*a*). As an initial approximation, the rod end is assumed to be hemispherical. Since the rod end is symmetrical about the vertical axis, the corresponding semicircle is divided into segments Fig. 1.12(*b*). Depending upon the accuracy requirements of the final result a suitable number of segments are selected. The circular arcs on the periphery are called the 'contour segment'. The procedure is explained as follows:

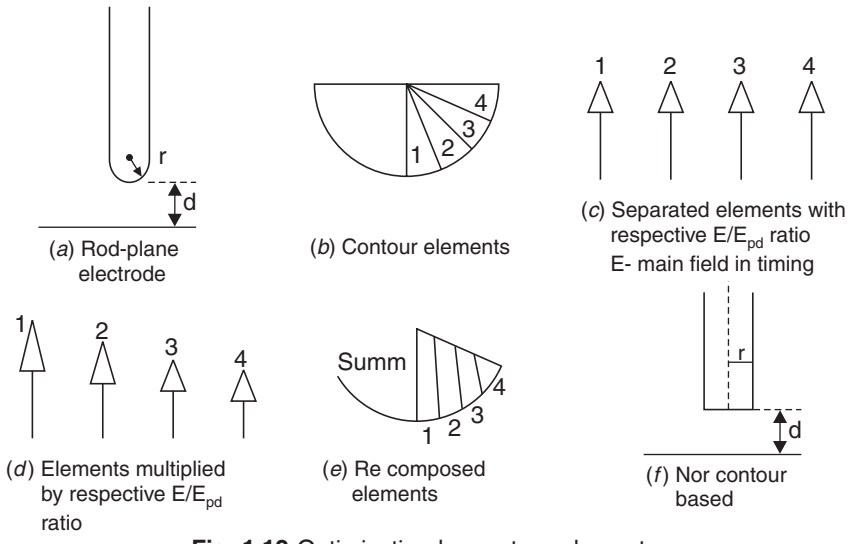


Fig. 1.12 Optimisation by contour elements

(*i*) At the outset, mean electric field intensity *E* on each contour element is calculated using any of the numerical techniques mentioned in article 1.8. Contour elements are shown separately with their corresponding magnitudes of E/E_{pd} ratio in Fig. 1.12(*c*).

(*ii*) The radius of each element which is same in this case as all elements lie along the arc of a circle is multiplied by their respective $(E/E_{pd})^n$ whereas the angles of the elements and therefore, the shapes remain unchanged. The exponent which represents no. of iterations required to converge, is recommended to be four. As a result of multiplication a new set of elements of different sizes is obtained shown in Fig. 1.12(*d*). The new set of elements are recomposed as shown in Fig. 1.12(*e*) thereby a new contour is obtained. The radius of the new contour is made equal to *r* by multiplying the radius of the new contour by a suitable scale factor.

(*iii*) The new contour is placed at the distance '*d*' from the plane as shown in Fig. 1.12(*f*). The steps (*i*) and (*ii*) are repeated until the remaining differences in E/E_{pd} ratio are sufficiently small. As seen in Fig. 1.12(*e*) during each iteration the surface region is flattened where E/E_{pd} exceeds its mean value whereas the remaining region becomes more curved which results into a more uniform distribution of E/E_{pd} . It is to be noted that in step (*ii*) if the E_{pd} depends upon the curvature of the electrode surface, then instead of the element radii, the reciprocal of the curvature is multiplied by $(E/E_{pd})^n$. It is to be noted with caution that the boundary conditions in terms of operating constraints if any must be included during the process of optimisation of electrode configuration otherwise it may turn out to an infeasible solution.

For further details a few references are given at the end of the chapter.

PROBLEMS

1.1. "The potential at a point plays an important role in obtaining any information regarding the electrostatic field at that point." Justify the statement.

1.2. Starting with Laplace's equation in two-dimension explain the Finite Difference Method for evaluation of field distribution. Discuss its advantages and disadvantages.

1.3. What is Finite Element Method for evaluation of field distribution? Discuss the procedure associated with this method and discuss its advantages and limitations.

1.4. What is a global stiffness matrix when referred to Finite Element Method used for evaluation of electric field ?

1.5. Discuss the basic philosophy associated with charge simulation method for evaluation of electric field distribution.

1.6. Obtain expressions for potential coefficients *p* for a (*i*) point charge (*ii*) line charge (*iii*) ring charge distribution.

1.7. What are field intensity coefficients when referred to charge simulation method?

1.8. Explain briefly in a few steps the procedure associated with charge simulation method. Discuss its advantages and limitations.

1.9. Explain briefly an improved version of charge simulation method and discuss its advantages and limitations.

1.10. Compare in brief various numerical techniques to study or evaluate the electric field distribution in dielectric medium.

1.11. Explain with neat diagrams the application of an Electrolytic tank for evaluating electric field distribution in dielectric medium or electrical equipments.

1.12. Explain with neat diagrams the procedure to control electric field intensity in high voltage equipments.

1.13. Explain clearly what do you mean by optimisation of Electrode configuration. Mention various techniques used for the purpose.

1.14. Discuss briefly the optimisation techniques listed here:

(*i*) Displacement of contour points with flow chart.

(*ii*) Changing the location of optimisation charges and contour points.

(*iii*) Modification of contour elements.

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